



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 32

Floater, Caps & Floors

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EXAMPLE

- Consider the barbell strategy consisting of longs in two bonds S & L with equal money weights (INR 1,000) of 1 year and 9 year ZCBs and short in a ZCB liability M of INR 4,022.71 at $t=5$ years. All the bonds are trading at a YTM of 15% p.a. Assume that there is a instantaneous non-parallel shift of the yield curve due to which we have $S_{01} = 15\%$, $S_{05} = 18\%$ & $S_{09} = 15\%$. Evaluate the performance of the strategy.

SOLUTION

- $P_S = \frac{1,000 \times 1.15}{1.15} = \frac{1,150}{1.15} = 1,000$
- $P_L = \frac{1,000 \times 1.15^9}{1.15^9} = \frac{3,517.88}{1.15^9} = 1,000$
- $P_{BARBELL} = P_S + P_L = 2,000$
- $P_M = \frac{4022.71}{1.18^5} = 1758.36$
- Portfolio Value: 241.64
- Further, larger the value of S_{05} greater will be the curvature of the yield-price curve, lesser will be P_M and larger will be the portfolio value.

CONDOR TRADES

- Condor trades work the same and are evaluated the same as butterfly trades.
- The only modification is that two positions with relatively close duration are used for the bullet.
- For example, a super bullet butterfly could be constructed as long the 7 duration and short the 3 and 11 duration wings.
- A condor is similar but could use an equal weighted combination of long the 6.5 and 7.5 duration instead of a single long 7 duration position.
- The wings would remain as short the 3 and 11 duration.



MISC DEBT INSTRUMENTS



CAPPED AND FLOORED FLOATERS



WHAT IS A FLOATER?

- A floating-rate bond (“floater”) pays a coupon that adjusts every period based on an underlying reference rate.
- The coupon is typically paid in arrears, meaning the coupon rate is determined at the beginning of a period but is paid at the end of that period.



FLOATING RATE BOND VALUATION



- Consider a floating rate bond with a notional principal of N currency units. Let us consider the valuation of the bond at an arbitrary time t . Let T_0 be the first reset date of the bond after t at which the floating rate for the period (T_0, T_1) is fixed. The first payment from the bond is made at T_1 .
- Thereafter, the rate is reset at T_1 for the period (T_1, T_2) and the next payment is made at T_2 and so on. The final payment is made at T_n which is the maturity of the bond.

- **Case 1: $t \leq T_0$. Consider the following strategy:**
- **At time t , buy N bonds each of face value 1 unit and maturity at T_0 for a price of $P(t, T_0)$.**
- **The cash outflow at time t is $NP(t, T_0)$.**
- **The cash inflow at time T_0 is N .**
- **We use this cash N at T_0 to buy $\frac{N}{P(T_0, T_1)}$ number of bonds of face value 1 and maturity T_1 costing $P(T_0, T_1)$ per bond.**
- **These bonds yield $\frac{N}{P(T_0, T_1)} = N \left(\frac{1}{P(T_0, T_1)} - 1 \right) + N$ at T_1 .**

- Out of this cash inflow, we use N to buy $\frac{N}{P(T_1, T_2)}$ bonds of unit face value maturing at T_2
- We are left with a surplus cash of $N \left(\frac{1}{P(T_0, T_1)} - 1 \right) = N \left(\frac{1 - P(T_0, T_1)}{P(T_0, T_1)} \right)$ which represents the floating rate interest at the rate fixed at T_0 on the principal N for the period (T_0, T_1) .

- Thus, this strategy completely replicates the cash flows from the floating rate bond.
- Hence, by no arbitrage the cost of this strategy must equal the value of the floating rate bond.
- Thus, value of floating rate bond at t is $P_{fl}(t) = NP(t, T_0)$.
- If $t = T_0$, then $P_{fl}(t) = NP(T_0, T_0) = N$

- **Case 2: $T_0 < t < T_1$ i.e. valuation of the floating rate bond between two reset dates.**
- **The cash flow at T_1 is receivable from the bond at T_1 includes the notional principal and the floating rate interest for the period (T_0, T_1) at the floating rate r_{01}^{fl} which is set at T_0 applicable for the period (T_0, T_1) .**
- **Hence, total cash flow at T_1 is $N \left(1 + r_{01}^{fl} \delta \right)$.**

- Thus, the no of unit face value bonds to be invested in at t is $N \left(1 + r_{01}^{fl} \delta \right)$ to replicate the floating rate bond's cash flow.
- Thus, value of replicating strategy $N \left(1 + r_{01}^{fl} \delta \right) P(t, T_1)$.

CAPPED FLOATER

- A capped floater effectively contains an issuer option that prevents the coupon rate from rising above a specified maximum rate known as the cap.
- Value of a capped floater = Value of a “straight” floater – Value of the embedded cap.

FLOORED FLOATER

- A related floating rate bond is the floored floater, which has a coupon rate that will not fall below a specified minimum rate known as the floor.
- In this instance, the embedded option belongs to the investor and offers protection from falling interest rates.
- Value of a floored floater = Value of a “straight” floater + Value of the embedded floor

VALUATION OF CAPPED & FLOORED FLOATERS

- We can use the standard backward induction methodology in a binomial interest rate tree to value a capped or floored floater.
- As with the valuation of a bond with embedded options, we must adjust the value of the floater at each node to reflect the exercise of an in-the-money option (in this case, a cap or a floor).



EXAMPLE

- Consider a \$100 par, two-year, floating-rate note that pays LIBOR (set in arrears). The underlying bond has the same credit quality as reflected in the LIBOR swap curve. The two year binomial tree is given.

One-period forward rate	
Year 0	Year 1
4.5749%	7.1826%
	5.3210%

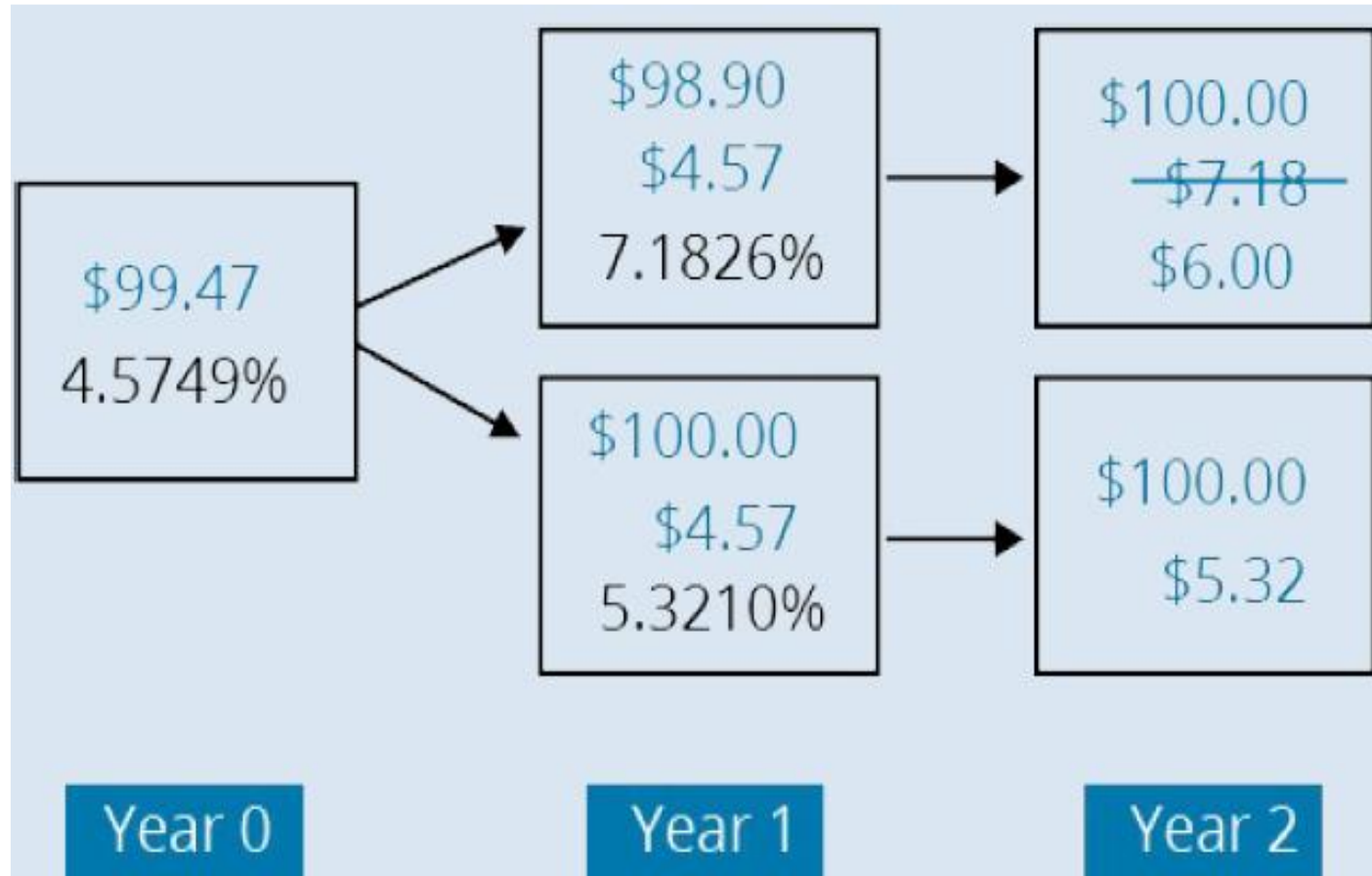
- **Compute the following?**
 1. **The value of the floater, assuming that it is an option-free bond.**
 2. **The value of the floater, assuming that it is capped at a rate of 6%. Also compute the value of the embedded cap.**
 3. **The value of the floater, assuming that it is floored at a rate of 5%. Also compute the value of the embedded floor.**

SOLUTION

- 1. An option-free bond with a coupon rate equal to the required rate of return will be worth par value.**
- 2. Hence, the straight value of the floater is \$100.**



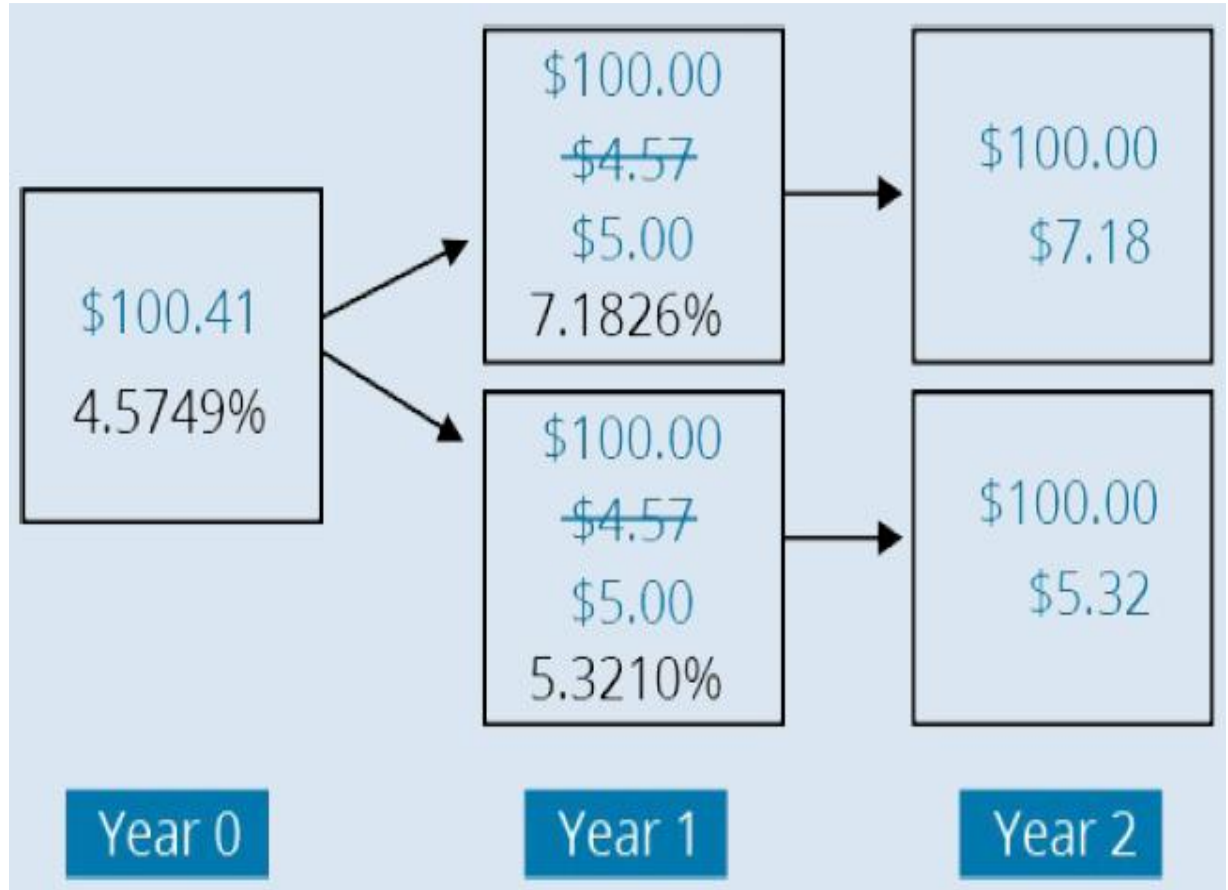
2. The value of the capped floater is as shown:



- The upper node in year 2 shows the exercise of the cap (the coupon is capped at \$6.00 instead of rising to \$7.18).
- Note that when the option is not in the money, the floater is valued at par.
- $V_{1,U} = (\$100 + \$6) / (1 + 0.071826) = \$98.90$
- $V_{1,L} = (100 + 5.3210) / (1 + 0.05321) = \$100$

- The year 0 value is the average of the year 1 values (including their adjusted coupons) discounted for one period. In this case, the year 1 coupons require no adjustment, as the coupon rate is below the cap rate.
- $$V_0 = \frac{1}{2} \times \frac{[(98.90+4.57)+(100+4.57)]}{(1.045749)} = \$99.47$$
- Thus the value of the embedded cap
- $= \$100 - \$99.47 = 0.53.$

- 3. The value of the floored floater is \$100.41, as shown below:



- The nodes for year 2 show the coupons for that period (none of the rates are below the floor, and hence the floor is not exercised).
- Strikethroughs for both nodes in year 1 indicate that the floor was in the money; we replace the LIBOR-rate coupon with a coupon based on the floor strike rate of 5%.
- The year 0 value is the average of the year 1 values (including their adjusted coupons) discounted for one period:
- $$V_0 = \frac{1}{2} \times \frac{(100+5)+(100+5)}{(1.045749)} = \$100.41$$
- Thus the value of the embedded floor
- $= \$100.41 - \$100 = \$0.41.$

EXAMPLE

- Consider the three bonds:
- Bond X, a 3%, 15-year option-free bond.
- Bond Y, a 3%, 15-year callable bond.
- Bond Z, a 3%, 15-year puttable bond.
- Which bond's embedded option is most likely to increase in value if the yield curve flattens?
- If the yield curve flattens, the forward rates are likely to fall. This will induce the issuer to call back the bonds.
- Thus, when an upward sloping yield curve flattens, call options increase in value while put options decrease in value.

EXAMPLE

- A callable bond returned an OAS of 145 bp using a binomial tree with a volatility of 15%. If the volatility is now increased to 19%, what will happen to the OAS?
- When the assumed volatility in a binomial tree increases, the value of the embedded option increases while the value of the straight bond does not change. Thus, the computed value of OAS will decrease.

EXAMPLE

- Consider the 3 bonds:
- Bond X, a 3%, 15-year option-free bond.
- Bond Y, a 3%, 15-year callable bond.
- Bond Z, a 3%, 15-year puttable bond.
- Which bond is most likely to have the highest effective duration?
- **Straight bonds generally have higher effective durations than bonds with embedded options. Both call and put options have the potential to reduce the life of a bond, so the duration of callable and puttable bonds will be less than or equal to that of their straight counterparts.**

EXAMPLE

- Which of the following is correct?
 - I. The highest key rate duration of a low-coupon callable bond corresponds to the bond's time-to-exercise.
 - II. The highest key rate duration of a high-coupon puttable bond corresponds to the bond's time-to-maturity.
- Statement I is incorrect. Low-coupon callable bonds are unlikely to be called; hence, their highest key rate duration corresponds to their time-to-maturity.
- Statement II is correct. High coupon puttable bonds are unlikely to be put and hence, their highest key rate duration corresponds to their time-to-maturity.

EXAMPLE

- Which bond of the following:
- Bond X, a 3%, 15-year option-free bond.
- Bond Y, a 3%, 15-year callable bond.
- Bond Z, a 3%, 15-year puttable bond.
- is least likely to experience the highest increase in value, given a parallel downward shift of 150 bps in the yield curve?
- The price appreciation for a callable bond due to decline in interest rate is limited due to the call feature; callables exhibit negative convexity at low rates. Hence a decline in rates is least likely to result in best price performance for a callable bond.

EXAMPLE

- Which bond of the following is most likely to experience an increase in effective duration due to an increase in interest rates?
- Bond X, a 3%, 15-year option-free bond.
- Bond Y, a 3%, 15-year callable bond.
- Bond Z, a 3%, 15-year puttable bond.
- When interest rates increase, a callable bond becomes less likely to be called (its duration will increase).
- The put option in a puttable bond would be more likely to be exercised in a rising interest rate scenario and hence, the duration of a puttable bond would decrease. Duration of an option-free bond would also decrease as interest rate increases (but not as significantly).

EXAMPLE

The OAS for two very similar callable corporate bonds are as given. Which one is overpriced?

Bond	OAS
A	46 bps
B	34 bps

The OAS reflects the credit risk of the corporate callable bond over Treasuries, since option risk has been removed.

Relative to bond A, bond B has lower OAS. Given that the two bonds have similar credit risk, bond B offers a lower OAS for the same level of risk as bond A and thus would be considered overpriced.